Q1

Find the lowest common denominator by multiplying $(y+3)$ by $(y-6)$

lowest common denominator is $(y+3)(y-6)$

[1]

Multiply top-and-bottom of each fraction by the brackets needed to form this denominator

$$\frac{2(y-6)}{(y+3)(y-6)} - \frac{(y+3)}{(y+3)(y-6)}$$

Write as one single fraction over the common denominator
Subtract the numerators

$$\frac{2(y-6)-(y+3)}{(y+3)(y-6)}$$

[1]

Expand the brackets in the numerator

$$\frac{2y-12-y-3}{(y+3)(y-6)}$$

Collect "like" terms in the numerator

$$\frac{y-15}{(y+3)(y-6)}$$ [1]

Q2

2

Find the lowest common denominator by multiplying $(x-3)$ by $(x+3)$

lowest common denominator is $(x-3)(x+3)$

[1]

Multiply top-and-bottom of each fraction by the brackets needed to form this denominator

$$\frac{5(x+3)}{(x-3)(x+3)} - \frac{4(x-3)}{(x-3)(x+3)}$$

Write as one single fraction over the common denominator
Subtract the numerators

$$\frac{5(x+3)-4(x-3)}{(x+3)(x-3)}$$

[1]

Expand the brackets in the numerator

$$\frac{5x+15-4x+12}{(x+3)(x-3)}$$

Collect "like" terms in the numerator

$$\frac{x+27}{(x+3)(x-3)}$$ [1]

Q3

Find the lowest common denominator by multiplying 2 by 3

lowest common denominator is $2 \times 3 = 6$

[1]

Multiply top-and-bottom of each fraction by the numbers needed to form this denominator

$$\frac{3(x+1)}{6} + \frac{2(x+3)}{6}$$

Write as one single fraction over the common denominator
Add the numerators

$$\frac{3(x+1)+2(x+3)}{6}$$

[1]

Expand the brackets in the numerator

$$\frac{3x+3+2x+6}{6}$$

Collect "like" terms in the numerator

$\frac{5x+9}{6}$ [1]

Q4-5

Factorise the denominator (for example, by finding two numbers that multiply to give -15 and add to give 2)
One factor is likely to be $(x+5)$

$$\frac{4(x+5)}{(x+5)(x-3)}$$

[1]

Cancel any common brackets from top-and-bottom

$$\frac{4\cancel{(x+5)}}{\cancel{(x+5)}(x-3)}$$

$\frac{4}{x-3}$ [1]

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Write out the denominator as the product of two brackets

$$\frac{3(x+1)}{(x+1)(x+1)}$$

Cancel the common brackets from top-and-bottom

$$\frac{3\cancel{(x+1)}}{(x+1)\cancel{(x+1)}}$$

$\frac{3}{x+1}$ [1]

Q6

6

First we need the lowest common denominator (lcd) of x, $2x$ and 4.

$$\text{lcd of } x,\ 2x \text{ and } 4 = 4x$$

Rewrite each fraction over the lcd.

$$\frac{3}{x} = \frac{3\times 4}{4x} = \frac{12}{4x}$$
$$\frac{x+2}{2x} = \frac{(x+2)\times 2}{4x} = \frac{2x+4}{4x}$$
$$\frac{1}{4} = \frac{1\times x}{4x} = \frac{x}{4x}$$

[1]

Now add the fractions by adding the numerators over a the single (lcd) denominator.

$$\frac{12}{4x} + \frac{2x+4}{4x} + \frac{x}{4x} = \frac{12+2x+4+x}{4x}$$

[1]

Collect like terms (on the numerator) and simplify if possible.

$$\frac{12+2x+4+x}{4x} = \frac{3x+16}{4x}$$

This cannot be simplified.

$$\therefore \frac{3}{x} + \frac{x+2}{2x} + \frac{1}{4} = \frac{3x+16}{4x}$$ [1]

Q7

7

First we need the lowest common denominator (lcd) of $x-2$ and $x+1$.
Since these share no common factors, the lcd will be their product.

$$\text{lcd of } x-2 \text{ and } x+1 = (x-2)(x+1)$$

Rewrite each fraction over the lcd.

$$\frac{4}{x+2} = \frac{4(x+1)}{(x-2)(x+1)}$$
$$\frac{3}{x-1} = \frac{3(x-1)}{(x-2)(x+1)}$$

[1]

Now subtract the fractions by subtracting the numerators over the denominator (lcd).

$$\frac{4}{(x-2)} - \frac{3}{(x+1)} = \frac{4(x+1)-3(x-2)}{(x-2)(x+1)}$$

Expand the brackets on the numerator and collect like terms.

$$\frac{4(x+1)-3(x-2)}{(x-2)(x+1)} = \frac{4x+4-3x+6}{(x-2)(x+1)}$$
$$= \frac{x+10}{(x-2)(x+1)}$$

[1]

This cannot be simplified.

$$\therefore \frac{4}{x-2} - \frac{3}{x+1} = \frac{x+10}{(x-2)(x+1)}$$ [1]

Q8

8

Find the lowest common denominator by multiplying 4 by 3

lowest common denominator is $4 \times 3 = 12$

Multiply top-and-bottom of each fraction by the numbers needed to form this denominator

$$\frac{3(2x+1)}{12} + \frac{4(x-2)}{12}$$

[1]

Write as one single fraction over the common denominator
Add the numerators

$$\frac{3(2x+1)+4(x-2)}{12}$$

Expand the brackets in the numerator

$$\frac{6x+3+4x-8}{12}$$

[1]

Collect "like" terms in the numerator

$$\frac{10x-5}{12}$$

Check for anything that can be factorised and/or cancelled
In this case the numerator has a factor of 2

$$\frac{5(2x-1)}{12}$$ [1]

Q9

9

First we need the lowest common denominator (lcd) of $3x$, $5x$ and $10x$.
All already have a factor of x, so focus on the 3, 4 and 5. (5 and 10 both have a factor of 5)

$$\text{lcd of } 3x,\ 5x \text{ and } 10x = 30x$$

Rewrite each fraction over the lcd.

$$\frac{2}{3x} = \frac{2 \times 10}{30x} = \frac{20}{30x}$$
$$\frac{4}{5x} = \frac{4 \times 6}{30x} = \frac{24}{30x}$$
$$\frac{9}{10x} = \frac{9 \times 3}{30x} = \frac{27}{30x}$$

[1]

Now add/subtract the fractions over a the single (lcd) denominator.

$$\frac{20+24-27}{30x} = \frac{17}{30x}$$

This cannot be simplified.

$$\therefore \frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x} = \frac{17}{30x}$$ [1]

Q10-11

10

Both the numerator and denominator are quadratic expressions.
These can be factorised but notice in particular that the denominator is the difference of two squares.

$$\frac{10x^2+23x+12}{4x^2-9}=\frac{(5x+4)(2x+3)}{(2x+3)(2x-3)}$$

Factorising numerator [1]
Factorising denominator [1]

There is a factor of $(2x+3)$ on both numerator and denominator so these cancel.

$$\frac{5x+4}{2x-3}$$ [1]

11

Both fractions already have a common denominator.
So we only need add the numerators.

$$\frac{3}{x}+\frac{4}{x}=\frac{3+4}{x}$$
$$=\frac{7}{x}$$

The first option should be circled.

$\frac{7}{x}$ $\frac{7}{2x}$ $\frac{12}{x}$ $\frac{12}{x^2}$

[1]

Q12

12

To multiply fractions, we multiply numerators, then denominators.
If we can spot any common factors on the. numerator and denominator first, whether they be letters or numbers, the multiplication can be easier.

25 and 5 share a common factor 5
2 and 8 share a common factor 2

$$\frac{\overset{5}{\cancel{25}a}}{\underset{4}{\cancel{8}}}\times\frac{\overset{1}{\cancel{2}a}}{\underset{1}{\cancel{5}}}$$

Be careful if there is lots of crossing out and if it helps, rewrite the simplified fractions, then multiply.

$$\frac{5a}{4}\times a=\frac{5a^2}{4}$$

This cannot be simplified and is a single fraction.

$$\therefore \frac{25a}{8}\times\frac{2a}{5}=\frac{5a^2}{4}$$

Correct single fraction but not in its simplest form [1]
Fully correct [1]

You can multiply first and simplify after if you prefer.

Q13

Fractions need a common denominator before they can be subtracted.
$3x$ and $2x$ already share a factor of x, so their lowest common denominator (lcd) will come from finding 3×2.

lcd of $3x$ and $2x$ is $6x$

Now rewrite the fractions with the common denominator.

$$\frac{x+4}{3x} - \frac{5}{2x} = \frac{2(x+4)}{6x} - \frac{3 \times 5}{6x}$$

One correct fraction [1]
Both correct [1]

Now we can write this as a single fraction, subtracting the numerators.

$$\frac{2(x+4)}{6x} - \frac{3 \times 5}{6x} = \frac{2(x+4) - 15}{6x}$$

Expand and simplify the numerator.

$$\frac{2(x+4) - 15}{6x} = \frac{2x + 8 - 15}{6x}$$
$$= \frac{2x - 7}{6x}$$

Nothing else can be factorised/simplified/cancelled.

$\frac{x+4}{3x} - \frac{5}{2x} = \frac{2x-7}{6x}$, i.e. $a = 2$, $b = -7$, $c = 6$ [1]

Q14

14

Before simplifying the fraction, look to see if the numerator or denominator can be factorised.
In this case the denominator can be.

$$\frac{3x^2}{6x^2 + 3} = \frac{3x^2}{3(2x^2 + 1)}$$

We can now see there is a factor of 3 on both the numerator and denominator so these will cancel.

$$\frac{3x^2}{3(2x^2 + 1)} = \frac{x^2}{2x^2 + 1}$$

Nothing else can be simplified.
The third option should be circled.

$\frac{x^2}{2x^2 + 3}$ $\frac{x^2}{6x^2 + 1}$ **$\frac{x^2}{2x^2 + 1}$** $\frac{1}{2} + x^2$

[1]

Q15

15

Find the lowest common denominator by multiplying $(x-1)$ by $(x+2)$.

lowest common denominator is $(x-1)\times(x+2)=(x-1)(x+2)$

Multiply top-and-bottom of each fraction by the factor needed to form this denominator.

$$\frac{3(x+2)}{(x-1)(x+2)}+\frac{4(x-1)}{(x-1)(x+2)}$$

[1]

Write as one single fraction over the common denominator.
Add the numerators.

$$\frac{3(x+2)+4(x-1)}{(x-1)(x+2)}$$

[1]

Expand the brackets in the numerator.

$$\frac{3x+6+4x-4}{(x-1)(x+2)}$$

Collect "like" terms in the numerator.

$$\frac{7x+2}{(x-1)(x+2)}$$

There is nothing else that factorises or cancels.

$$\therefore \frac{3}{x-1}+\frac{4}{x+2}=\frac{7x+2}{(x-1)(x+2)}$$ [1]

Q16

16

Find the lowest common denominator by multiplying $(n+1)$ by n.

lowest common denominator is $(n+1)\times n=n(n+1)$

Multiply top-and-bottom of each fraction by the factor needed to form this denominator.

$$\frac{(m+1)n}{n(n+1)}-\frac{m(n+1)}{n(n+1)}$$

Write as one single fraction over the common denominator.
Subtract the numerators.

$$\frac{n(m+1)-m(n+1)}{n(n+1)}$$

[1]

Expand the brackets in the numerator.

$$\frac{mn+n-mn-m}{n(n+1)}$$

Collect "like" terms in the numerator.

$$\frac{n-m}{n(n+1)}$$

Nothing else factorises or cancels.

$$\therefore \frac{m+1}{n+1}-\frac{m}{n}=\frac{n-m}{n(n+1)}$$ [1]